Chirosolitons: Unique Spatial Solitons in Chiral Media

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Abstract—**We show analytically that unique spatial solitons (chirosolitons) can propagate in chiral media that exhibit Kerr-type nonlinearities. In contrast to the solitons in achiral media, unique features of the chirosolitons can be seen in their elliptically polarized nature, the bisoliton state, and the possibility of superluminous phase propagation. Conditions for supporting the bright- and dark-type chirosolitons are discussed.**

I. Introduction

THE SOLITON phenomenology has now become ubiquitous in contemporary science and engineering [1]. Of these, the type that can be described by the cubic nonlinear Schrödinger equation (NLSE) [2] is regarded as one of the most attractive research objects. A representative example of this soliton is found in the laser beam (or pulse) propagation in Kerr-type nonlinear media. More recently, signals that show evidence for the microwave soliton have been detected [3] by using intensified magnetostatic-wave propagation in a ferromagnetic thin film. As is well known, there exist two kinds of solitons in the canonical (1+1)-dimensional NLSE: bright and dark solitons. We predict in this paper the existence of a new vectorial soliton, termed a chirosoliton, which could be observed for intense electromagnetic-beam propagation in chiral media [4]-[10] with Kerr nonlinearities. Chiralities are a topological nature of objects, which are definable with the lack of any translational and rotational symmetry between themselves and their mirror images [4]. Representative examples are found in helical conductors, Möbius strips, and organic polymers. Chiral media for electromagnetic-wave propagation are realizable by embedding a number of chiral objects in achiral host media [5]. Recent progress in polymeric engineering permitted one to obtain chiral media that appear useful for wide range of radiation spectra. Only recently, nonlinear optical experiments concerning the second-harmonic generation through a fourth-order nonlinear susceptibility due to molecular chirality [11] and the optical Kerr shutter using the optical rotatory dispersion of (+)-haxahelicene ([6]) 4% CS_2 solution [12] have been reported. Chilarity is currently an important issue in liquid crystal physics [13], [14]. In this context, we note that liquid crystals often exhibit exceptionally large intensity-dependent nonlinearities [15]. However, except the harmonic generation, previous studies concerning the electromagnetic chirality were to our knowledge focused upon its ability to manipulate the polarization vector [4]–[10], [12], the basic principle of which is similar to optical activities. Our finding in this paper suggests that the combined use of the chirality and the Kerr nonlinearity (what we call "chiral nonlinearity" [12], [16]) can give rise to electromagnetic spatial solitons with unique features that cannot be found in conventional nonlinear-Schrödinger-type solitons. In particular, in addition to their polarization properties (i.e., the elliptically polarized nature), unique features of the chirosolitons can be seen in the existence of two branches (bisolitons) for the fundamental soliton solution and the possibility of the superluminous (the fast-wave) propagation for their wave front. Conditions for supporting the bright- and dark-type chirosolitons are discussed.

II. Basic Equations

We consider nonlinear electromagnetic-wave propagation in an isotropic Kerr host medium in which a number of chiral guest objects are densely embedded to enhance chirality. For such chiral media the constitutive equations with respect to the relevant electromagnetic-field vectors are well documented in the literature, and are given by [4]–[10]

$$D = \epsilon E + i\xi B, \quad H = i\xi E + \mu^{-1} B \tag{1}$$

where E, H, D, and B, are the electric-field, the magnetic-field, the electric-displacement, and the magnetic-flux-density vectors, respectively; ϵ, μ, and ξ are the permittivity, the permeability, and the chiral admittance, respectively, of a chiral medium under consideration. For achiral media ($\xi = 0$) one recovers the usual constitutive relations. We assume here that the permittivity is intensity dependent (Kerr type). Note that in recent years, not to mention the optical nonlinearity, the dielectric nonlinearity in the microwave and millimeter-wave regions was also investigated both experimentally [17], [18] and theoretically [19]. Also assumed is that the imaginary part of the chiral admittance is negligible [6]–[12], [16]. Depending on the helicity of a chiral medium, the chirality admittance can be positive or negative. Of course, no modification is required on Maxwell's equations. In what follows, we concentrate on the stationary wave, and then all the field vectors are assumed to be factorized into the laterally dependent amplitude and a propagation phase factor as $A(x)\exp[i(\beta z - \omega t)]$, where $A(x)$ represents the lateral distribution of $A(x, z, t)$; the symbol A represents E, H, D, and B. No variation along the transverse (y) axis is considered, i.e., throughout the following algebra we retain $\partial/\partial y \equiv 0$. On substitution of (1) into source-free Maxwell's equations, coupled wave equations of the

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lateral electromagnetic-field components, $E_x(x)$ and $H_x(x)$, are derivable

$$-Z_0\xi d(\tilde{\epsilon}_r^{-1} dE_x/dx)/dx + Z_0\xi(k_0^2\mu_r + \beta^2\tilde{\epsilon}_r^{-1})E_x + d(\tilde{\epsilon}_r^{-1}G_x/dx)/dx + (k_0^2\mu_r - \beta^2\tilde{\epsilon}_r^{-1})G_x = 0, \quad (2a)$$

$$-(\mu_r Z_0\xi)^{-1}d^2E_x/dx^2 - Z_0\xi d(\tilde{\epsilon}_r^{-1}dE_x/dx)/dx - [k_0^2\tilde{\epsilon}_r(Z_0\xi)^{-1} + k_0^2\mu_r Z_0\xi - \beta^2(\mu_r Z_0\xi)^{-1} - \beta^2\tilde{\epsilon}_r^{-1}Z_0\xi]E_x + d(\tilde{\epsilon}_r^{-1}dG_x/dx)/dx - (k_0^2\mu_r + \beta^2\tilde{\epsilon}_r^{-1})G_x = 0 \quad (2b)$$

where $G_x \equiv iZ_0H_x$, Z_0 is the intrinsic impedance of vacuum, k_0 is the wavenumber in vacuum ($k_0 = \omega/c_0$, where ω is the angular frequency, and c_0 is the speed of light in vacuum), β is the phase constant along the z axis, $\tilde{\epsilon}_r$ is the relative permittivity (the tilde denotes a nonlinear quantity), and μ_r is the relative permeability. Here we imply $\epsilon_r \geq 1$, $\mu_r \geq 1$.

Note that apparently (2) are too complicated to treat them analytically. Although, with the aid of a numerical method, direct analysis of them is not necessarily impossible, it requires much computational effort. To render an analytical approach possible, below we adopt a reasonable assumption that the induced variation of the nonlinear permittivity $\tilde{\epsilon}_r$ is considerably smaller than the linear part ϵ_r. With this assumption, additional terms that contain derivatives of $\tilde{\epsilon}_r$ can be dropped, and we obtain a reduced version of (2)

$$-Z_0\xi d^2E_x/dx^2 + Z_0\xi(k_0^2\tilde{\epsilon}_r\mu_r + \beta^2)E_x + d^2G_x/dx^2 + (k_0^2\tilde{\epsilon}_r\mu_r - \beta^2)G_x \cong 0, \quad (3a)$$

$$-[\tilde{\epsilon}_r(\mu_r Z_0\xi)^{-1} + Z_0\xi]d^2E_x/dx^2 - [k_0^2\tilde{\epsilon}_r^2(Z_0\xi)^{-1} + k_0^2\tilde{\epsilon}_r\mu_r Z_0\xi - \beta^2\tilde{\epsilon}_r(\mu_r Z_0\xi)^{-1} - \beta^2 Z_0\xi]E_x + d^2G_x/dx^2 - (k_0^2\tilde{\epsilon}_r\mu_r + \beta^2)G_x \cong 0. \quad (3b)$$

In what follows we consider the mode with $G_x = \eta E_x$ (where η is a dimensionless constant). Substituting this relation into (3), we obtain

$$d^2E_x/dx^2 + [(\eta + Z_0\xi)(\eta - Z_0\xi)^{-1}k_0^2\tilde{\epsilon}_r\mu_r - \beta^2] \cdot E_x = 0 \quad \text{for } \eta \neq Z_0\xi, \quad (4a)$$

$$d^2E_x/dx^2 + \{[\tilde{\epsilon}_r\mu_r^{-1} + (Z_0\xi)^2 + \eta Z_0\xi] \times [\tilde{\epsilon}_r\mu_r^{-1} + (Z_0\xi)^2 - \eta Z_0\xi]^{-1}k_0^2\tilde{\epsilon}_r\mu_r - \beta^2\} \cdot E_x = 0 \quad \text{for } \tilde{\epsilon}_r\mu_r^{-1} + (Z_0\xi)^2 - \eta Z_0\xi \neq 0. \quad (4b)$$

Comparison between these two equations results in

$$(\eta + Z_0\xi)(\eta - Z_0\xi)^{-1} = [\tilde{\epsilon}_r\mu_r^{-1} + (Z_0\xi)^2 + \eta Z_0\xi] \cdot [\tilde{\epsilon}_r\mu_r^{-1} + (Z_0\xi)^2 - \eta Z_0\xi]^{-1}$$

from which, after brief algebra, we obtain

$$Z_0\xi[\eta^2 - \tilde{\epsilon}_r\mu_r^{-1} - (Z_0\xi)^2] = 0. \quad (5)$$

Solving this for η we can derive the explicit form of the constant η

$$\eta_\pm \equiv \pm[\tilde{\epsilon}_r\mu_r^{-1} + (Z_0\xi)^2]^{1/2} \quad \text{for } \xi \neq 0$$
$$\cong \pm[\epsilon_r\mu_r^{-1} + (Z_0\xi)^2]^{1/2}. \quad (6)$$

The physical meaning of each sign will be mentioned later. Note that (6) is meaningful solely for nonvanishing ξ; for $\xi = 0$, (5) holds identically. (In the limit of $\xi \to 0$, (3) are decoupled, and $\eta_\pm$ do not have to be zero). With (6), (4a), and (4b) become compatible each other, and degenerate to single equation; for the isotropic Kerr-type nonlinearity [20]

$$\tilde{\epsilon}_r = \epsilon_r + \kappa(|E_x|^2 + |E_y|^2 + |E_z|^2) \quad (7)$$

it can be written explicitly in the form

$$d^2E_x/dx^2 + [f_\pm(\epsilon_r, \mu_r, \xi)k_0^2\epsilon_r\mu_r - \beta^2 + f_\pm(\epsilon_r, \mu_r, \xi)k_0^2\mu_r\kappa(|E_x|^2 + |E_y|^2 + |E_z|^2)]E_x = 0 \quad (8)$$

with

$$f_\pm(\epsilon_r, \mu_r, \xi) \equiv \mu_r\epsilon_r^{-1}(Z_0\xi + \eta_\pm)^2, \quad (9a)$$

$$E_y = i\, v_p\mu(Y_0\eta_\pm + \xi)E_x, \quad (9b)$$

$$E_z = i\,\beta^{-1}\partial E_x/\partial x \quad (9c)$$

where the coefficient κ governs the magnitude of the nonlinearity, $v_p = \omega/\beta$, and $Y_0 = 1/Z_0$. From experimental results concerning the Kerr-type chiral nonlinearity [12], [16], it would be reasonable to understand that, at least for such weak nonlinearities as assumed in this paper, the constitutive relations (1) used in isotropic chiral media remain valid for the chiral Kerr media. From (6) and (9a) $f_\pm$ is constantly positive. Because (8) is a one-dimensional cubic NLSE, it can predict the two kinds of soliton solutions: the bright and the dark solitons.

III. Soliton Solutions

As an ansatz of the fundamental bright soliton we set

$$E_x(x) = A_0 \text{ sech } (\alpha x) \quad (10)$$

where A_0 and α are real constants that represent soliton amplitude and a reciprocal beam width, respectively. From (9b), (9c), and (10), (8) can be reduced to the form

$$d^2E_x/dx^2 + [f_\pm(\epsilon_r, \mu_r, \xi)k_0^2\epsilon_r\mu_r - \beta^2 + f_\pm(\epsilon_r, \mu_r, \xi)k_0^2\mu_r\kappa_\pm|E_x|^2]E_x = 0 \quad (11)$$

with

$$\kappa_\pm \equiv \{1 + k_0^2\mu_r[\epsilon_0^{-1}\mu(Y_0\eta_\pm + \xi)^2 + (A_0^2\kappa f_\pm/2)]\beta^{-2}\}\kappa$$
$$\cong [1 + k_0^2\mu_r\epsilon_0^{-1}\mu(Y_0\eta_\pm + \xi)^2\beta^{-2}]\kappa \quad (12)$$

where a fourth-order term that depends on $|E_x|^4$, which arises from the $|E_z|^2$ term, has been dropped as we focus in this paper solely on the lowest-order (i.e., the Kerr) nonlinearity. From (10) and (11) we obtain

$$\alpha = k_0A_0[(\mu_r\kappa_\pm/2)f_\pm(\epsilon_r, \mu_r, \xi)]^{1/2}, \quad (13a)$$

$$\beta = k_0\{f_\pm(\epsilon_r, \mu_r, \xi)\mu_r[\epsilon_r + (\kappa_\pm/2)A_0^2]\}^{1/2}. \quad (13b)$$

The intensity FWHM can be calculated by the relation: FWHM $= 1.7627/\alpha$. Because α must be real, the Kerr coefficient κ should be positive (self-focusing). On substitution of (12) into (13b) one obtains a quadratic of β^2. This is solvable for β to yield finally

$$\beta_\pm \equiv k_0\{f_\pm(\epsilon_r, \mu_r, \xi)\mu_r[\epsilon_r + (\kappa'/2)A_0^2]\}^{1/2} \quad (14)$$

where $\kappa' \equiv (1+Y_0)\kappa$. In the derivation of (14) we have neglected higher-order, additional terms with order of $(\kappa A_0^2)^2$. Note that in the limit of the vanishing chirality ($\xi \to 0$), $f_\pm \to 1$, and eventually (10) with (13a) and (14) reduces to that for the usual bright soliton. It is found from (13a) and (14) that both the spot size and the propagation constants are dependent on $f_\pm$. In particular, the variation of the phase constant $\beta_\pm$ against the magnitude of chirality appears interesting since $\beta_\pm$ determines the phase velocity (v_p) of the chirosoliton through the relation $v_p = \omega/\beta_\pm$. From (9a) and (14) one can obtain the following inequalities

$$A_0^2 < 2(\epsilon_r\kappa')^{-1}[-\epsilon_r^2 + \epsilon_r\mu_r^{-1} - 2Z_0\xi + 2(Z_0\xi)^2] \cdot \text{for } 0 < \beta_\pm < k_0 \text{ (fast-wave region)} \quad (15a)$$

$$A_0^2 > 2(\epsilon_r\kappa')^{-1}[-\epsilon_r^2 + \epsilon_r\mu_r^{-1} - 2Z_0\xi + 2(Z_0\xi)^2] \cdot \text{for } \beta_\pm > k_0 \text{ (slow-wave region).} \quad (15b)$$

It should be noted here that the fast wave permits of superluminous propagation in the sense that the phase velocity of the soliton exceeds the speed of light in vacuum, i.e., $v_p > c_0$. To be superluminous, it follows from (15a) that

$$\xi > (Y_0/2)[1 + (2\epsilon_r^2 - 2\epsilon_r\mu_r^{-1} + 1)^{1/2}] \quad (16a)$$

or

$$\xi < (Y_0/2)[1 - (2\epsilon_r^2 - 2\epsilon_r\mu_r^{-1} + 1)^{1/2}]. \quad (16b)$$

Note that at least with $\epsilon_r \geq 1$, $\mu_r \geq 1$, the argument of the square root on the right-hand side of (16) remains positive. Numerical results for (16) are shown in Fig. 1, where it is assumed that $\mu_r = 1$. For instance, for $\epsilon_r \sim 1.5$, $\mu_r \sim 1$, we obtain from (16b), $\xi \lesssim -0.77$ mS. We find that although this value of $|\xi|$ is substantially larger than that of typical optically active media, it is comparable to those considered in the literature with regard to the electromagnetic chirality [6]-[10]. In addition, the use of a liquid crystal in the cholesteric phase may permit of a chilarity comparable to this value. This superluminous wave propagation, by no means, contradicts the causality and the special theory of relativity, because we restrict our attention to the phase velocity [21]. Indeed, in a waveguide bounded by a perfect conductor, it has been well known that the phase velocity may exceed c_0. Evidently, the situation is essentially different from the superluminous soliton we have discovered here where the physical system is nonlinear and unbounded (open). It should be emphasized that the possibility of the superluminous bright-soliton formation is indeed a unique feature of the present chirosoliton. Aside from the dark soliton, superluminous phase propagation is, in principle, never achievable with the use of the conventional bright soliton in achiral media.

We would like to stress here that such superluminous solitons are indeed physically reasonable and, by no means,

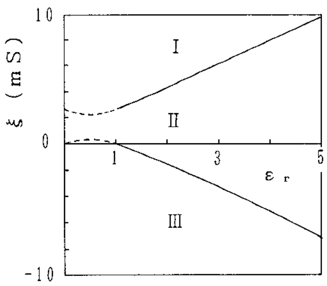

Fig. 1. Regions for superluminous (I and III) and subluminous (II) phase propagation. The chiral admittance ξ of (16) is plotted versus the relative permittivity ϵ_r. The relative permeability μ_r is set to be unity. The two superluminous (fast-wave) regions are marked with I for (16a) and III for (16b). The gap region marked with II offers the subluminous (the slow-wave) region.

arise from any mathematical artifice. They can be explained intuitively as follows. We first note that to support bright solitons the linear phase terms in the left-hand side of (11) should be negative, i.e., the following inequality

$$f_\pm(\epsilon_r, \mu_r, \xi)k_0^2\epsilon_r\mu_r - \beta^2 < 0$$

must hold. Consider the limit of $\xi \to 0$ (achiral limit). In this limit, as derived from (9a) with (6), $f_\pm \to 1$. Thus the inequality is reduced to $k_0^2\epsilon_r \times \mu_r - \beta^2 < 0$, i.e., $\beta^2 > k_0^2\epsilon_r\mu_r$, which verifies that without chirality, superluminous phase propagation is not, in principle, achievable. However, with chirality being included ($\xi \neq 0$) the situation is expected to change radically. Since $f_\pm$ are the explicit function of ξ, with careful choice of the chirality parameter the magnitude of $f_\pm$ could be varied in an optional fashion. Specifically, for $f_\pm < 0$, superluminous phase propagation is unconditionally possible. As already shown in (16) and Fig. 1, through algebra and numerical calculations we have ensured that this is indeed the case. On the other hand, for dark solitons to be supported, the linear phase terms in (11) should be positive, i.e., the inequality

$$f_\pm(\epsilon_r, \mu_r, \xi)k_0^2\epsilon_r\mu_r - \beta^2 > 0$$

must hold. It is obvious that in sharp contrast to the case of bright solitons, even in the limit of $\xi \to 0$, i.e., $f_\pm \to 1$, they can become superluminous ($\beta^2 < k_0^2\epsilon_r\mu_r$).

Subsequently we discuss the dark-type chirosoliton. As an ansatz of the black-type dark soliton we set

$$E_x(x) = A_0 \tanh(\alpha x). \quad (17)$$

From (9b), (9c), and (17), (8) can be reduced to the form that is identical to (11). From (11) and (17) we obtain

$$\alpha = k_0A_0[-(\mu_r\kappa_\pm/2)f_\pm(\epsilon_r, \mu_r, \xi)]^{1/2} \quad (18a)$$

$$\beta = k_0[f_\pm(\epsilon_r, \mu_r, \xi)\mu_r(\epsilon_r + \kappa_\pm A_0^2)]^{1/2}. \quad (18b)$$

In contrast to the bright soliton, from (12) and (18a) the Kerr coefficient κ should be negative (self-defocusing). On substitution of (12) into (18b) one obtains a quadratic of β^2, which yields

$$\beta_{\pm U} = k_0[f_\pm(\epsilon_r, \mu_r, \xi)\mu_r(\epsilon_r + \kappa' A_0^2)]^{1/2}, \quad (19a)$$

$$\beta_{\pm L} = k_0 A_0[-f_\pm(\epsilon_r, \mu_r, \xi)\mu_r \kappa Y_0]^{1/2} \quad (19b)$$

where the subscripts 'U' and 'L' indicate, respectively, the upper and the lower branches, both of which are found to be oscillatory; as in the bright soliton we have dropped additional terms with order of $(\kappa A_0^2)^2$. Note that in the limit of the vanishing chirality, (17) with (18a) and (19a) is reduced to that for the conventional black soliton. From (9a) and (19a) we obtain

$$A_0^2 > (\epsilon_r \kappa')^{-1}[-\epsilon_r^2 + \epsilon_r \mu_r^{-1} - 2Z_0\xi + 2(Z_0\xi)^2] \quad \cdot\text{for } 0 < \beta_{\pm U} < k_0 \text{ (fast-wave region)} \quad (20a)$$

$$A_0^2 < (\epsilon_r \kappa')^{-1}[-\epsilon_r^2 + \epsilon_r \mu_r^{-1} - 2Z_0\xi + 2(Z_0\xi)^2] \quad \cdot\text{for } \beta_{\pm U} > k_0 \text{ (slow-wave region).} \quad (20b)$$

It should be noted that for field intensities that meet (20a) the dark-soliton propagation can become superluminous ($v_p > c_0$). However, in contrast to the bright soliton, this condition can in principle be met even for the vanishing chirality [set $\xi = 0$ in (20a)]. A unique feature we would like to stress here is the fact that the dark chirosoliton ($\xi \neq 0$) should propagate in a superluminous fashion independently of the beam intensity, provided that the chirality lies in the range that is identical to (16). From (9a) and (19b) the intensity relation of the lower branch, the existence of which is unique to the chirosoliton, is obtainable

$$A_0^2 < (-f_\pm \mu_r \kappa Y_o)^{-1} \text{ for } 0 < \beta_{\pm L} < k_0 \quad \cdot\text{(fast-wave region)} \quad (21a)$$

$$A_0^2 > (-f_\pm \mu_r \kappa Y_0)^{-1} \text{ for } \beta_{\pm L} > k_0 \quad \cdot\text{(slow-wave region).} \quad (21b)$$

We shall check here whether in the limit of vanishing nonlinearity ($\kappa \to 0$) one recovers the right(+)- and left(−)-handed elliptically-polarized plane waves that are known to propagate in linear chiral media [6]–[10]. In this limit, with $\partial/\partial x \equiv 0$, the nontrivial condition for (11) predicts the two phase constants

$$k_\pm \equiv k_0(\epsilon_r \mu_r f_\pm)^{1/2} = \pm\omega\mu\xi + [\epsilon_r \mu_r k_0^2 + (\omega\mu\xi)^2]^{1/2} \quad (22)$$

where (9a) has been substituted. We find that these coincide with those presented in the literature [6]–[10].

IV. Polarization Property

Finally, we discuss the polarization (the vectorial) characteristic of the chirosoliton. From Maxwell's equations with the constitutive relations (1), all the electromagnetic-field components are expressible in terms of E_x

$$H_x = -iY_0\eta_\pm E_x \quad (23a)$$

$$H_y = v_p(\epsilon + Y_0\eta_\pm \mu\xi + \mu\xi^2)E_x \quad (23b)$$

$$H_z = Y_0\eta_\pm \beta^{-1}\partial E_x/\partial x \quad (23c)$$

$$D_x = (\epsilon + Y_0\eta_\pm \mu\xi + \mu\xi^2)E_x \quad (24a)$$

$$D_y = iv_p\mu[(\epsilon + 2\mu\xi^2)(Y_0\eta_\pm + \xi) + \epsilon\xi]E_x \quad (24b)$$

$$D_z = i\beta^{-1}(\epsilon + Y_0\eta_\pm \mu\xi + \mu\xi^2)\partial E_x/\partial x \quad (24c)$$

$$B_x = -i\mu(Y_0\eta_\pm + \xi)E_x \quad (25a)$$

$$B_y = v_p\mu[\epsilon + 2\mu\xi(Y_0\eta_\pm + \xi)]E_x \quad (25b)$$

$$B_z = \mu\beta^{-1}(Y_0\eta_\pm + \xi)\partial E_x/\partial x. \quad (25c)$$

Note that the results of E_y and E_z have been given in (9b) and (9c), respectively. It can be seen from these equations that H_y, D_x, and B_y are in phase with E_x, whereas E_y, E_z, H_x, D_y, D_z, and B_x are in quadrature with it. Also seen is that all the longitudinal components (E_z, H_z, D_z, and B_z) exhibit a parity that is opposite to the remaining [the lateral (x) and the transverse (y)] components. From (9b) the ellipticity of the polarization vector is given by $|E_y/E_x| = v_p\mu|Y_0\eta_\pm + \xi|$. Here it is obvious from (9b) that for $Y_0\eta_\pm + \xi > 0$, one predicts the left-handed elliptically-polarized state, whereas for $Y_0\eta_\pm + \xi < 0$, one predicts the right-handed counterpart. For instance, with $\eta_- < 0$ [the lower sign of (6)] and $\xi < 0$ [(16a)], the trajectory of the electric-field vector should be right-handed. Since, as seen in (6) and (14) [or (19)], both $\eta_\pm$ and v_p are a function of ξ, the ellipticity will depend on the chirality in a nontrivial fashion. A numerical example will be shown elsewhere.

V. Conclusion

We have presented a novel type of spatial soliton, termed a chirosoliton, which arises from the combined effect of chirality and nonlinearity of electromagnetic media. Subsequently we have predicted some unique properties of this soliton such as the possibility of superluminous phase propagation and the existence of two (upper and lower) branches in the dark soliton solution. Moreover, we have discussed its polarization (vectorial) characteristics. As was predicted for conventional electromagnetic (microwave and optical) solitons in achiral media, exploring exotic and unusual solitonlike entities, such as multidimensional solitons (quasisolitons) [22], [23] including localization in both space and time, dark soliton crosses [24], [25], electromagnetic vortices [26], [27], bright-dark symbions [28], and bright-kink symbions [29], is of great interest as a future research topic.

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[21] Since in this paper we do not consider pulse (wave packet) propagation, we cannot mention whether the group and/or the energy velocities could also become superluminous. However, it should be noted here that as for the phase velocity, in certain media, superluminosity of the group and the energy velocities does not conflict with causality. For more details, see, e.g., R. Y. Chiao, "Superluminal (but causal) propagation of wave packets in transparent media with inverted atomic populations," *Phys. Rev. A*, vol. 48, pp. R34-R37, July 1993.

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